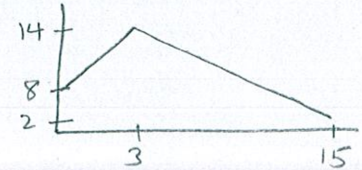
A person's velocity (in meters per second) at time t (in seconds) is given by $v(t) = \begin{cases} 2t+8, & 0 \le t \le 3 \\ 17-t, & 3 \le t \le 15 \end{cases}$. SCORE: _____ / 5 PTS

[a] Find the exact distance the person travelled from time $t = 0$ seconds to $t = 15$ seconds.
NOTE: You must show the arithmetic expression that you used to get your answer.
You may only use techniques discussed in sections 5.1 and 5.2.



$\frac{1}{2}$ $\frac{8+14}{2} \cdot 3 + \frac{14+2}{2} \cdot 12$ $\frac{1}{2}$

$= 11 \cdot 3 + 8 \cdot 12$

$= 33 + 96 = 129$ METERS

$\frac{1}{2}$ $\frac{1}{2}$

[b] Estimate the distance the person travelled from time $t = 0$ seconds to $t = 15$ seconds using three subintervals and left endpoints.
NOTE: You must show the arithmetic expression that you used to get your answer.

$\Delta t = \frac{15-0}{3} = 5$

$v(0)\Delta t + v(5)\Delta t + v(10)\Delta t$

$= 8 \cdot 5 + 12 \cdot 5 + 7 \cdot 5$

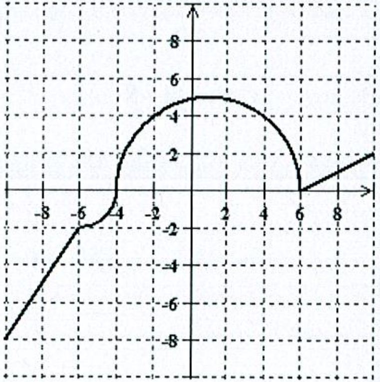
$= (8+12+7) \cdot 5$

$1\frac{1}{2}$ $\frac{1}{2}$

$= 27 \cdot 5 = 135$ METERS

$\frac{1}{2}$ $\frac{1}{2}$

The graph of function f is shown on the right. **SCORE: _____ / 4 PTS**



<u>The graph consists of a diagonal line, arcs of 2 circles, then another diagonal line.</u>

[a] Evaluate $\int_{-10}^{10} f(x)\,dx$.

<u>NOTE: You must show the arithmetic expression that you used to get your answer.</u>

$$\frac{1}{2}\ \underbrace{\frac{1}{2}\pi(5)^2}\ + \underbrace{\frac{1}{2}4\cdot 2}_{\frac{1}{2}} - \underbrace{\frac{8+2}{2}\cdot 4}_{\frac{1}{2}} - \underbrace{\frac{1}{4}\pi(2)^2}_{\frac{1}{2}}$$

$$= \frac{25}{2}\pi + 4 - 5\cdot 4 - \pi$$

$$= \underbrace{\frac{23}{2}\pi - 16}\ \frac{1}{2}$$

[b] Evaluate $\int_{6}^{-10} f(x)\,dx$.

$$-\int_{-10}^{6} f(x)dx = -\left[\frac{25}{2}\pi - 5\cdot 4 - \pi\right] = \underbrace{-(}_{1}\underbrace{\frac{23\pi}{2} - 20)}_{\frac{1}{2}} = 20 - \frac{23\pi}{2}$$

Using the limit definition of the definite integral, and right endpoints, find $\int_{-2}^{1}(2x^2+8x)\,dx$. **SCORE: _____ / 10 PTS**

NOTE: Solutions using any other method will earn 0 points.

$$\Delta x = \frac{1--2}{n} = \frac{3}{n}$$

1 $\lim_{n\to\infty}\sum_{i=1}^{n} f\left(-2+\frac{3i}{n}\right)\frac{3}{n}$ 1

$= \lim_{n\to\infty}\frac{3}{n}\sum_{i=1}^{n}\left[2\left(-2+\frac{3i}{n}\right)^2+8\left(-2+\frac{3i}{n}\right)\right]$ $1\frac{1}{2}$

$= \lim_{n\to\infty}\frac{3}{n}\sum_{i=1}^{n}\left[2\left(4-\frac{12i}{n}+\frac{9i^2}{n^2}\right)-16+\frac{24i}{n}\right]$

$= \lim_{n\to\infty}\frac{3}{n}\sum_{i=1}^{n}\left[8-\cancel{\frac{24i}{n}}+\frac{18i^2}{n^2}-16+\cancel{\frac{24i}{n}}\right]$

$= \lim_{n\to\infty}\frac{3}{n}\sum_{i=1}^{n}\left[-8+\frac{18i^2}{n^2}\right]$ 2

$= \lim_{n\to\infty}\frac{3}{\cancel{n}}\left[-8\cancel{n}+\frac{\overset{3}{\cancel{18}}}{n^2}\,\frac{\cancel{n}(n+1)(2n+1)}{\cancel{6}}\right]$ 1 $\frac{1}{2}$ 1

$= \lim_{n\to\infty} 3\left[-8+3\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)\right]$ 1

$= 3[-8+3\cdot1\cdot2]$

$= 3(-2)$

$= -6$ 1 ONLY IF YOU USED THE $\lim_{n\to\infty}\sum_{i=1}^{n}$ DEFINITION + METHOD

Evaluate $\int_0^5 (|x-4| - 8\sqrt{25-x^2})\,dx$ using the properties of definite integrals and interpreting in terms of area. **SCORE: _____ / 5 PTS**

NOTE: You must show the proper use of the properties of the definite integral, NOT just the arithmetic.

$$\underbrace{\int_0^5 |x-4|\,dx - 8\int_0^5 \sqrt{25-x^2}\,dx}_{2} \overset{\frac{1}{2}}{=} \underbrace{\frac{1}{2}\cdot 4\cdot 4}_{} + \underbrace{\frac{1}{2}\cdot 1\cdot 1}_{\frac{1}{2}} - \underbrace{8\cdot\frac{1}{4}\pi(5)^2}_{1\frac{1}{2}}$$

$$= \underbrace{\frac{17}{2} - 50\pi}_{\frac{1}{2}}$$